10-2 Logarithms and Logarithmic Functions

y

$y = 2^x$

$(3, 8)$
$(4, 16)$

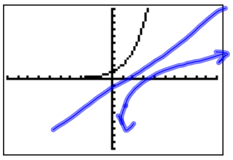
Find inverse

$x = 2^y$

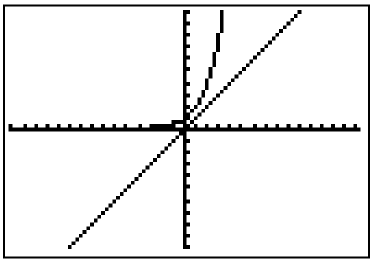
$\boxed{y = \log_2 x}$

$\log_2 8 = 3$

$\log_2 16 = 4$



Does this graph have an inverse?



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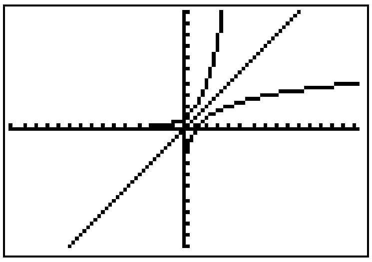

$y = 2^x$ $y = \log_2 x$ "log base 2 of x"

Inverses of each other

$y = b^x$ $y = \log_b x$

What is the inverse?

$y = 10^x$ $y = \log_{10} x$

Characteristics of a Logarithmic Function p532

1. The function is continuous and one-to-one.
2. The domain is the set of all positive real numbers.
3. The y-axis is an asymptote of the graph.
4. The range is the set of all real numbers.
5. The graph contains the point (1, 0). That is, the x-intercept is 1.

Suppose b and x are positive, and b ≠ 1, then, there is a number y such that:

$\log_b x = y$ iff $b^y = x$

"If and only if"

(Used to convert between logarithmic and exponential form.)

$\log_b x = y$ iff $b^y = x$

Logarithmic Form	Exponential Form
$\log_2 16 = 4$	$2^4 = 16$
$\log_2 8 = 3$	$2^3 = 8$
$\log_2 1 = 0$	$2^0 = 1$
$\log_2 x = y$	$2^y = x$
$\log_b x = y$	$b^y = x$

$\log_b x = y$ iff $b^y = x$

Logarithmic Form	Exponential Form
$\log_{10} 1000 = 3$	$10^3 = 1000$
$\log_{16} 4 = .5$	$16^{\frac{1}{2}} = 4$
$\log_3 27 = 3$	$3^3 = 27$
$\log_9 81 = 2$	$9^2 = 81$
$\log_5 25 = 2$	$5^2 = 25$
$\log_3 243 = 5$	$3^5 = 243$

Evaluate a logarithmic expression.

ex
$\log_2 64 = 6$

$\log_2 2^6$

6

$\log_2 64 = y$

$2^y = 64$

$2^y = 2^6$

6

ex
$\log_{25} 5 = y$

$\frac{1}{2}$

$25^y = 5$

$5^{2y} = 5^1$

$y = \frac{1}{2}$

ex
$\log_{10} 0.1$

$\log_{10} \frac{1}{10}$

-1

Remember:
Two functions are inverses iff
$[f \circ g]\, x = x$ and $[g \circ f]\, x = x$

Inverses of each other

$y = b^x \quad y = \log_b x$

$f(x) = b^x \quad g(x) = \log_b x$

$[f \circ g](x) = x \quad [g \circ f](x) = x$

$b^{\log_b x} = x$

$\log_b b^x = x$

Properties of logs

ex
$\log_2 2^5$

5

ex
$\log_4 4^2$

2

ex
$\log_7 49 = y$

$7^y = 49$

$7^y = 7^2$

2

ex
$3^{\log_3 5}$

5

ex
$8^{\log_8 10}$

10

DO:

1. $\log_{\frac{1}{2}} 32$ $= -5$ $\frac{1}{2}^{y} = 32$ $2^{-y} = 2^5$

2. $\log_9 27$ $= y$ $9^y = 27$ $3^{2y} = 3^3$ $y = \frac{3}{2}$

3. $\log_5 125$ $= 3$

4. $\log_8 4$ $= \frac{2}{3}$ $8^y = 4$ $2^{3y} = 2^2$

5. $9^{\log_9 5}$ $= 5$

6. $\log_{\sqrt{3}} 9\sqrt{3}$

p536
21-31odd
33-44